White-tailed Rising

Modeling Exponential Growth

MATERIALS

computer

Population Growth Generator Model

White-tailed deer (*Odocoileus virginianus*) are large mammals that are native to most of the U.S., Canada, Mexico, Central America, and South America. They are generalists and are capable of adapting to multiple types of habitats. Their sizes differ depending on their distance from the equator (Bergmann's Rule) and are found to be larger in colder environments and smaller in more tropical ones. White-tailed deer have few natural predators, which include wolves, cougars, jaguars, and American alligators.

Conservation efforts of the last century have allowed deer populations to increase drastically, causing their population sizes to exceed their carrying capacity in many areas. Deer may then venture into urban areas and become nuisance animals to farmers and reforestation efforts. Deer can also serve as vectors for tick-borne illnesses such as Lyme disease.

Typically, in late October or November female deer (does) will begin estrus mainly due to a decrease in the amount of daylight (photoperiod). Does can give birth to between 1 to 3 fawns per year. Depending on population size, does will mature sometime between 1 and 2 years. However, bucks (male deer) will mature and start breeding at about 1.5 years old.

In this experiment, you will simulate a population of white-tailed deer. Traditionally, this activity is performed with dice. Each die represents a single individual and the outcome of a dice roll determines the life event, such as a birth or death, for that individual. For example, with a starting population of 20 (20 dice), every "three" or "six" rolled represents the birth of an offspring, thereby adding an individual to your initial population. Each "one" rolled is a death, decreasing your initial population by one. Each roll of the dice represents one year in the population. Adding the numbers of births and subtracting the number of deaths from your initial population determines the size of subsequent populations.

You will be calculating populations over a series of "years," adding births and subtracting deaths from your initial population, until you finally reach 30 generations (years). The twist here is that we will be using a random number generator as part of a spreadsheet application instead of rolling hundreds of dice.

PURPOSE

In this activity, you will use the "Population Growth Generator Model" (free spreadsheet) to simulate the exponential growth of a deer population. You will also get a chance to use an equation to predict the size of a population undergoing exponential growth given a number of factors. Finally, you will get a chance to test out your own questions using either the model or the formula.

PROCEDURE

PART I: UNRESTRICTED EXPONENTIAL GROWTH OF THE DEER POPULATION

1. Download the "Population Growth Generator Model" to a computer and save the file to the desktop. Be sure to enable macros when opening the file.

2. Make sure that you are on the "Population Growth Generator" tab at the bottom of the page. You should see the "Population Growth Generator Model," which includes the following items:

   - In the upper left corner, a tally of births and *r* (intrinsic rate of increase) for that particular generation. The intrinsic rate of increase (*r*) is calculated by subtracting the deaths from the births, which the tally box does for you.

   - A reset button, which is used to reset the model when you input a new initial population size.

   - An area that allows you to change the ratio of births to deaths. The default is a 2:1 ratio (births to deaths), which is designated by the "2 = Ratio" area. You have the option to change the ratio by changing the number from 2 to whatever you want the value to be. For instance, if you want a 3:1 ratio just replace the 2 with a 3. The model is able to simulate all the following ratios: 1:1, 2:1, 3:1, 4:1, and 5:1.

3. Your starting population for this part will be 20, which you will put in the box that says "Initial" (the default is set at 5). This will model 20 individual dice being rolled at once. A die has six sides, so the model will randomly select a number (1 to 6) for each individual in the population.

4. Rather than counting all the 1s, 3s, and 6s to see how the population size changes during this generator, we can let the model do that work for us. The model will tally up the numbers of 3s and 6s and put them in the "Births" cell in the upper left corner. It will do the same for the 1s, but will put this number in the "Deaths" cell.

5. After you have entered 20 into the "Initial" box, press the "Reset" button once. This will reset the model to begin at a population of 20. Select the "Table and Graphs" tab at the bottom to see that the starting population is indeed 20.

6. As you can see on this tab, every time you go to a new generation it will input the new population size in another line. This will allow you to make as many generations as you want, and have the model do the data collecting for you.

PROCEDURE (CONTINUED)

7. Now go back to the "Population Growth Generator" tab and press the "New Generation" button. If you want, you can click on the "Table and Graphs" tab and see that the new population size has been placed in the next generation (second generation).

8. Select the "New Generation" button repeatedly until you have 30 generations. You will have to keep checking the table under the "Table and Graphs" tab to see when you reach 30.

9. Now you will graph the data. Your teacher might have you use the graph provided in this lesson to graph the data by hand or use the graphs already set up to have the spreadsheet application graph it for you.

   a. If you are using a Mac, the instructions for using the graph are as follows:

      i. Go to the "Table and Graphs" tab.

      ii. Right-click on the center of the "White-tailed Deer" graph.

      iii. Click on "Select Data."

      iv. Click on the box with the red arrow inside of it next to the "Y values" box.

      Y values: ='Table and Graphs'!$C$3(

      v. Click on the first value of the population size data (20), then hold and drag (highlight) the rest of the data all the way to the population at 30 generations.

      vi. Press "Enter."

      vii. Click on the box with the red arrow in it next to the "Category (X) axis labels" box.

      Category (X) axis labels:
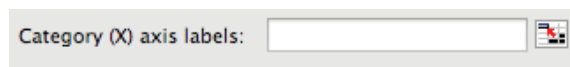
      viii. Highlight and drag the generation data (first generation to the last generation with which you started).

      ix. Press "Enter" and then click "OK." You should see the graph completed for you.

      x. Either print the graph or draw it on your student answer page.

PROCEDURE (CONTINUED)

b. If you are using a PC, the instructions for setting up the graph are as follows.

    i. Right-click on the center of the "White-tailed Deer" graph.

    ii. Click on "Select Data."

    iii. Click on the box with the red arrow in it next to the box at the top that says "Chart data range."

Chart data range: ='Exponetial Growth curve'!$C$4:$C$32

    iv. Highlight all of the population size values for each generation. Press "Enter."

    v. Click on the "Edit" button under the "Horizontal (Category) Axis Labels" title.

Horizontal (Category) Axis Labels

Edit

    vi. Click on the box with the red button in it next to the "Axis label range" box.

Axis label range:

    vii. Highlight all of the different generations (first generation to the last one measured).

    viii. Press "Enter" and then click "OK."

    ix. Click "OK" to accept all of the changes that you just made. You should see the graph completed for you.

    x. Either print the graph or draw it on your student answer page.

PROCEDURE (CONTINUED)

PART II: CALCULATING EXPONENTIAL GROWTH

With unlimited resources and zero deaths, bacteria will undergo exponential growth. Through binary fission, the population will continue to double in size with each "generation."

Go to the "Table and Graphs" tab on the Model. Under the bacterial growth column and starting with 1 bacterium in generation 1, enter the numbers that would coincide with each of 30 generations.

1. Enter 1 for the starting population next to Generation 1.

2. Click on the cell above this one. Enter the following formula into this cell:

   = [click on cell below] + [click on cell below]

   For instance, if the cell that contained the starting bacterium was in cell B30, then your formula would be

   =B30+B30

   Press "Enter." This will add the cell containing the 1 with itself, showing a 2 in the cell above it.

3. Click on the cell containing the 2. Now move your cursor to the bottom right corner of the cell until you see a "+" appear.

4. Click, hold, and drag the cursor up all the way to the last generation and then release it. This will drag the formatting (the formula that you just entered) to the rest of the cells above, allowing you to see the bacteria grow exponentially.

5. Follow the directions from Part I to graph the bacteria data, based on your teacher's instructions.

ANALYSIS

PART I: UNRESTRICTED EXPONENTIAL GROWTH OF THE DEER POPULATION

Use your data table to plot a graph demonstrating the unrestricted exponential growth of the deer population. Label the *lag phase* and *exponential phase* of your population.
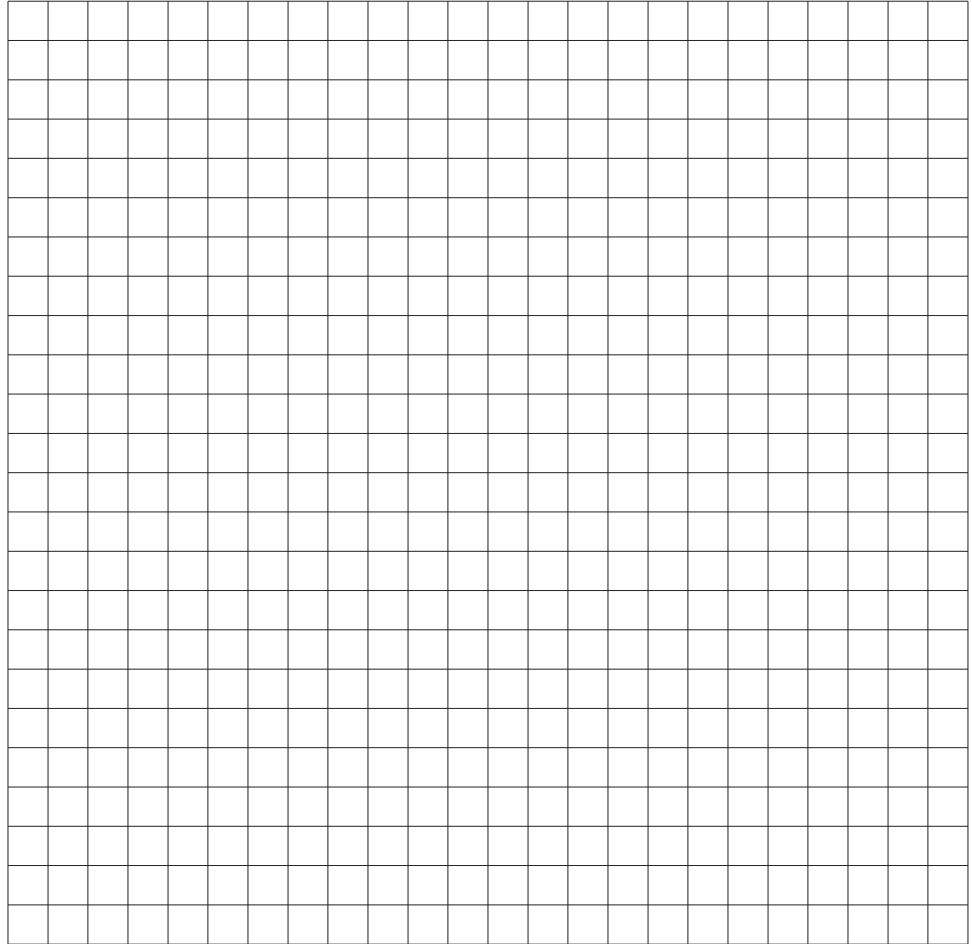
ANALYSIS (CONTINUED)

PART II: CALCULATING EXPONENTIAL GROWTH

We will now use an equation to make predictions of population size based on known variables.

$$X_{(t)} = a \cdot b^{t/\tau}$$ (Eq. 1)

where

$X_{(t)}$ = Population size at a specific time
a = Starting population
b = Growth factor
t = Total time allowed for growth
τ = Time it takes for X to increase by a factor of b

Say that one bacterium is allowed to undergo binary fission and reproduce for 12 generations. If we use Equation 1, we can determine how many bacteria will be in the population. The following are our known values for this problem:

a = Starting population (1)
b = Each generation results in a doubling of the population size (2)
t = Final number of generations (12)
τ = It takes one generation for the population to double (1)

Thus,

$$X_{(t)} = a \cdot b^{t/\tau} = 1 \cdot 2^{12/1} = 2^{12} = 4096 \text{ bacteria after 12 generations}$$

In the "Population Growth Generator Model," select the "Table and Graphs" tab. You should see that on the 13th generation (12 more after the first), there are 4096 bacteria in the population.

ANALYSIS (CONTINUED)

Use Equation 1 to solve the following scenarios:

1. One of the bacterial cultures that live in our intestines is *E. coli*. In a laboratory, the generation time for *E. coli* is 17 minutes. If you left a population of 20 and allowed it to reproduce for one day (24 hours), how many individual bacteria would be in the population?

2. *E. coli* makes up about 0.1% of our colon's contents. However, about 100 trillion bacteria are found in our intestines at any time. Based on your answer to Question 1, was your answer more or less than this amount? What prevents the bacteria in our intestines from attaining this enormous number?

ANALYSIS (CONTINUED)

3. Calculate the amount of deer that would be in the population after 40 years with a beginning population of 20. Assume that there are no deaths.

4. Calculate the amount of deer that would be in the population after 100 years. Assume there are no deaths.

CONCLUSION QUESTIONS

PART I: UNRESTRICTED EXPONENTIAL GROWTH OF THE DEER POPULATION

1. What is the ratio of births to deaths in this model population?

2. How many "years" did it take you to reach a population of 100?

3. After you reached a population of 100, how many more "years" did it take to reach a population of 200? How many more years to reach 300? 400? 500? Describe the pattern that you see based on your analysis.

4. Arithmetic growth can be typified by an increase of 20 deer per year. Draw a new line on your graph that represents this type of growth. Using data from the graph, describe the difference between arithmetic and exponential growth.

CONCLUSION QUESTIONS (CONTINUED)

5. Populations usually reach a carrying capacity (*k*) where the population becomes stable. There are two types of population limiting factors: density-dependent (dependent on the population size) and density-independent (not dependent on the population size).

   a. What are some examples of limiting factors that are density-independent? Describe their overall effect on a population's size.

   b. What are some examples of limiting factors that are density-dependent? Describe their overall effect on a population's size.

   c. Explain how these conditions would limit exponential growth and therefore allow carrying capacity to be reached.

CONCLUSION QUESTIONS (CONTINUED)

6. Figure 1 depicts how the human population growth has changed in the last 12,000 years. In 2012, Earth reached a population of 7.2 billion people. Answer the following questions using data from the graph.

Figure 1. Human population growth
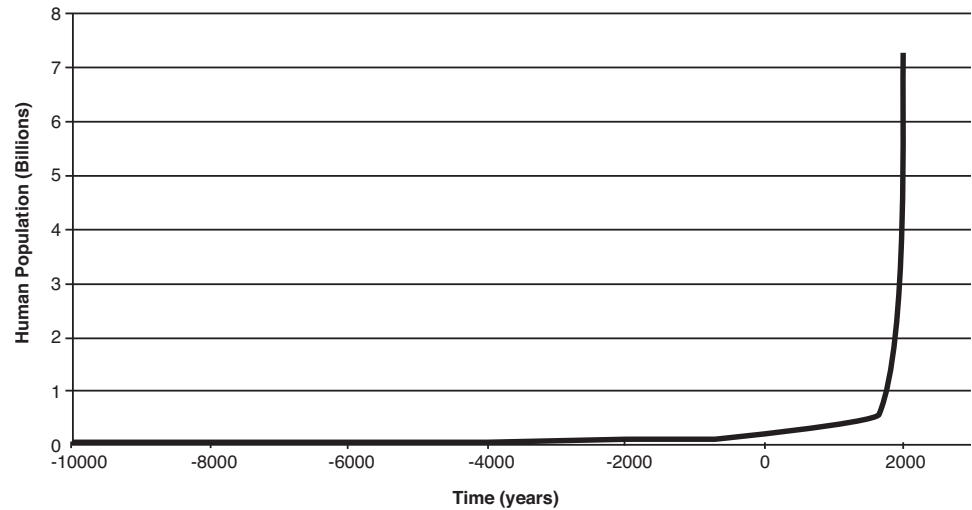
a. Approximately when does the human population increase dramatically?

b. What would you hypothesize for the reasons for this drastic increase in population size?

CONCLUSION QUESTIONS (CONTINUED)

PART II: CALCULATING EXPONENTIAL GROWTH

Answer the following questions using evidence from the graphs.

1. How do the two population growth curves differ?

2. What would account for these differences?

EXTENSION

Use the "Population Growth Generator Model" to ask your own questions regarding population growth. Following are some examples, but feel free to ask your own questions regarding populations of any species of your choice, and then use the Model to test your hypotheses.

- What if the ratio of births to deaths was 3:1 instead of 2:1?
- What would happen if the number of deaths equaled the number of births?
- What if the deer lived in an area that could only sustain a population of 500?
- After the tenth generation, a disease runs through a deer population that kills 75% of the population, and then the population is allowed to continue as before.
- Female mice can produce several litters per year (between 5 and 10), with each litter containing 5 or 6 babies. Each mouse reaches sexual maturity around four weeks of age. If two mice enter your home and start having babies, how many generations would it take to reach 1000 mice living in your home? 2000?